University of Kelaniya – Sri Lanka

Centre for Distance & Continuing Education

Bachelor of Science (General) External

First year First semester examination - 2019

(New Syllabus)

2022 August

Faculty of Science

APPLIED MATHEMATICS

Mechanics I – AMAT 16522

No.of Questions: 05(Five) **No.of Pages: Three(03)** **Time: Two(02)hrs**

Answer Four(04) questions only.

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01. (a) Define the inertial frames in Newtonian mechanics.

Prove that a frame which moves in a constant velocity with respect to an inertial frame is also an inertial frame.

(b) A stationary observer on Earth observes spaceships A and B moving in the same direction towards the Earth. Spaceship has speed $0.5c$ and spaceship B has speed $0.8c$, where c is the speed of light.

Determine the velocity of the spaceship A as measured by an observer at rest in spaceship B.

(c) Suppose the frame S is at rest and the frame $S'$is moving with uniform velocity v with respect to S, where $v < c$. Assume that the length of a measuring rod with respect to an observer in S is L and the length of it with respect to an observer who is at rest in S' is L'. Prove that

$$L = L'\left\{1 - \frac{v^2}{c^2}\right\}.$$

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A crew member of a spaceship measures the ship's length to be 100 m. The ship flies past Earth at a speed of 0.900 c. If an observer on Earth measures the length of the ship, what would he measure?

02. Obtain, in the usual notation, radial and transverse components of acceleration $\ddot{r} - r\dot{\theta}^2$ and $\frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}$, respectively in polar coordinates, of a particle moving in a plane.

A particle A of mass m moves on a smooth horizontal table and is connected by a light inextensible string passing through a smooth hole O on the table to a particle B of the same mass m which moves along the vertical through O. Initially B is at rest and A is distant a from O, moving with velocity $\sqrt{\frac{ga}{3}}$ perpendicular to OA.

Show that, in the subsequent motion,

(i) the distance $r = OA$ lies between a and $\frac{1}{2}a$.

(ii) tension in the string is $\frac{1}{6}mg\left(3 + \frac{a^3}{r^3}\right)$.

03. (a) Obtain the formulae

$$\frac{d^2u}{d\theta^2} + u = \frac{F(u)}{h^2u^2} \qquad \text{and} \qquad \dot{\theta} = hu^2$$

for the motion of the particle describing an orbit under an attraction P per unit mass, in the usual notation, where $u = \frac{1}{r}$.

If $f(u) = \mu\{u^2 - au^3\}$, where $a > 0$ and a particle is projected from an apse at a distance a from the center of force with a velocity $\left(\frac{\mu c}{a^2}\right)^{1/2}$, where $a > c$.

Prove that other apsidal distance of the orbit is $\frac{a(a+c)}{(a-c)}$.

04. (a) A particle is describing an ellipse of eccentricity e about a centre of force at a focus. Prove, in the usual notation, that

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \text{ and } h^2 = \mu a(1 - e^2).$$

A particle P, when it is at the end of the major axis closer to the focus O, meets and coalesces with an equal particle which is at rest. Prove that the composite particle describes an ellipse of eccentricity $\left(\frac{3-e}{4}\right)$.

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(b) under the influence of a force field, a particle of mass m moves along the ellipse $\underline{r} = a\cos\omega t\,\underline{i} + b\sin\omega t\,\underline{j}$.

If $\underline{p}$ is the momentum of the particle, prove that

(i) $\underline{r} \times \underline{p} = mab\omega\underline{k}$.

(ii) $\underline{r} \cdot \underline{p} = \frac{1}{2}m\omega(b^2 - a^2)\sin 2\omega t$.

05. (a) Assuming, in the usual notation, that $\frac{d\underline{A}}{dt} = \frac{\partial \underline{A}}{\partial t} + \underline{\omega} \times \underline{A}$ for a vector $\underline{A}$ with respect to two reference of frames, one rotating relative to the other, derive

$$\frac{d^2\underline{r}}{dt^2} = \frac{\partial^2\underline{r}}{\partial t^2} + \frac{\partial \omega}{\partial t} \times \underline{r} + 2\,\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} + \underline{\omega} \times (\underline{\omega} \times \underline{r}).$$

where $\underline{r}$ is the position vector of the particle with respect to the origin on the surface of the earth and $\underline{\omega}$ is the angular velocity of earth's diurnal rotation.

(b) Show that the position vector of a particle P projected from a point on the earth surface with velocity $\underline{u}$, with respect to a frame of reference with origin O on the surface of earth, and rotating with earth, is given by

$$\underline{r} = \underline{u}t + \frac{1}{2}\left(\underline{g} - 2\underline{\omega} \times \underline{u}\right)t^2 - \frac{1}{3}\left(\underline{\omega} \times \underline{g}\right)t^3$$

where $\underline{g}$ is the gravitational acceleration and $\underline{k}$ is the unit vector along the upward vertical through the point of projection. (If any assumption or approximation is used, it should be stated clearly)

A particle is projected westward with velocity v from a point on earth's surface whose northern latitude is λ. If the angle of projection is α, show that the particle takes the time

$\frac{v\sin\alpha}{g} - \frac{2\Omega v^2\cos\lambda\sin\alpha\cos\alpha}{g^2}$ to reach its maximum height h.

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